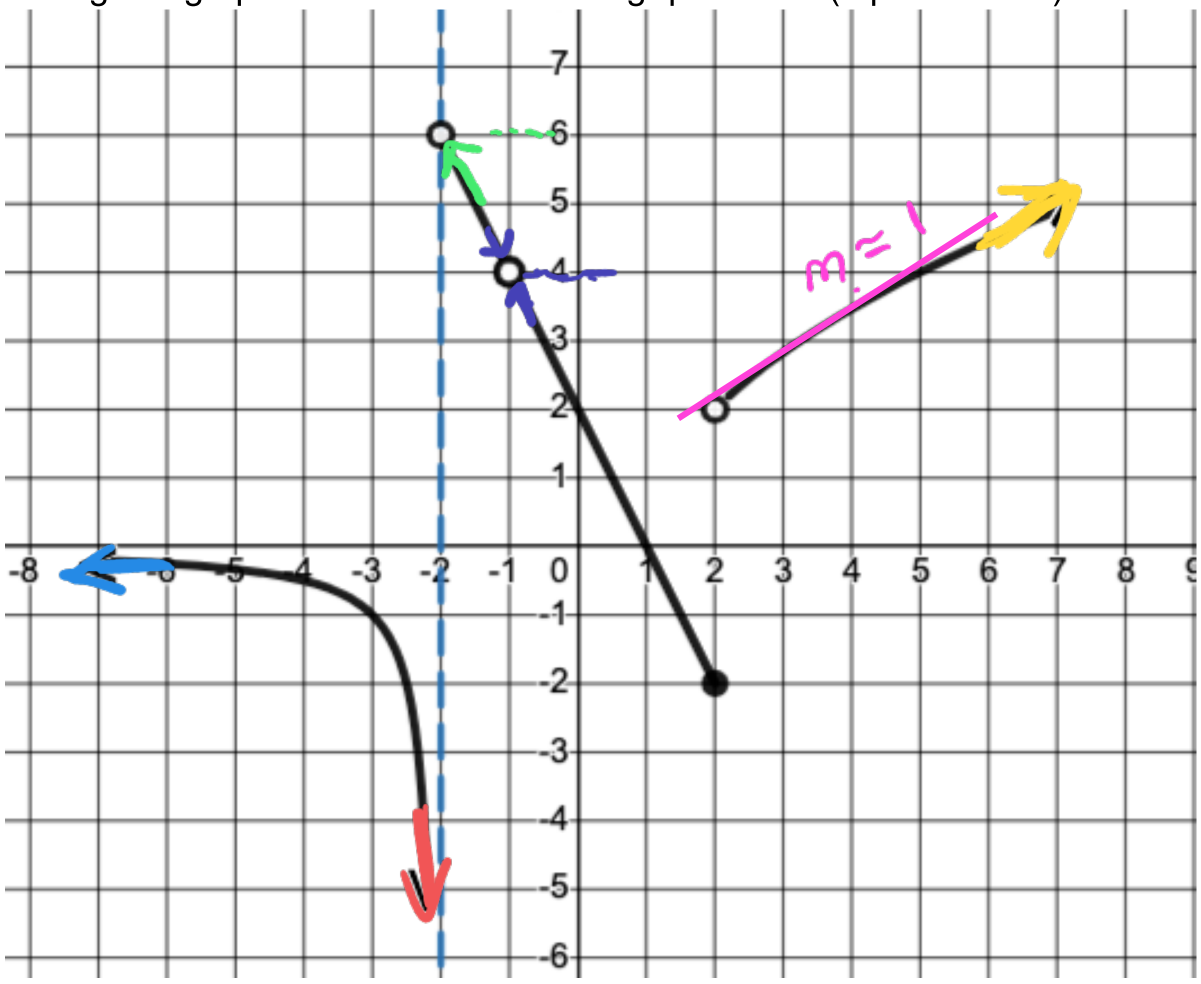
MATH-5A TEST 1 v2 (CHAPTER 1.4-1.8, 2.1, 3.4i and ii)
Spring 24

- **Detailed Instructions given in Canvas. Exact, simplified answers, good presentations with correct notation expected. Only methods learned in this class are allowed. All steps must be shown.**

(1) Use the given graph to answer the following questions. (2 points each)



(a) $\lim_{x\to -\infty} f(x)$ = 0.

(b) $\lim_{x\to -2^-} f(x)$ = $-\infty$.

(c) $\lim_{x\to -2^+} f(x)$ = 6.

(d) $\lim_{x\to \infty} f(x)$ = ∞.

(e) $f(-1)$ = undefined

(f) $\lim_{x\to -1} f(x)$ = 4

(g) Is $f(x)$ continuous at $x=-1$? NO

(h) Estimate $f'(4)$ 1

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(2) (a) Give the <u>formal</u> definition for $\lim_{x\to a^-} f(x)=\infty$ (8 points)

$a-\delta$ $a+\delta$ a

Given any $M>0$, there exists $\delta>0$ such that if $0-\delta<x<a$ $f(x)>M$

(b) Give the <u>formal</u> definition for $\lim_{x\to\infty} f(x)=L$

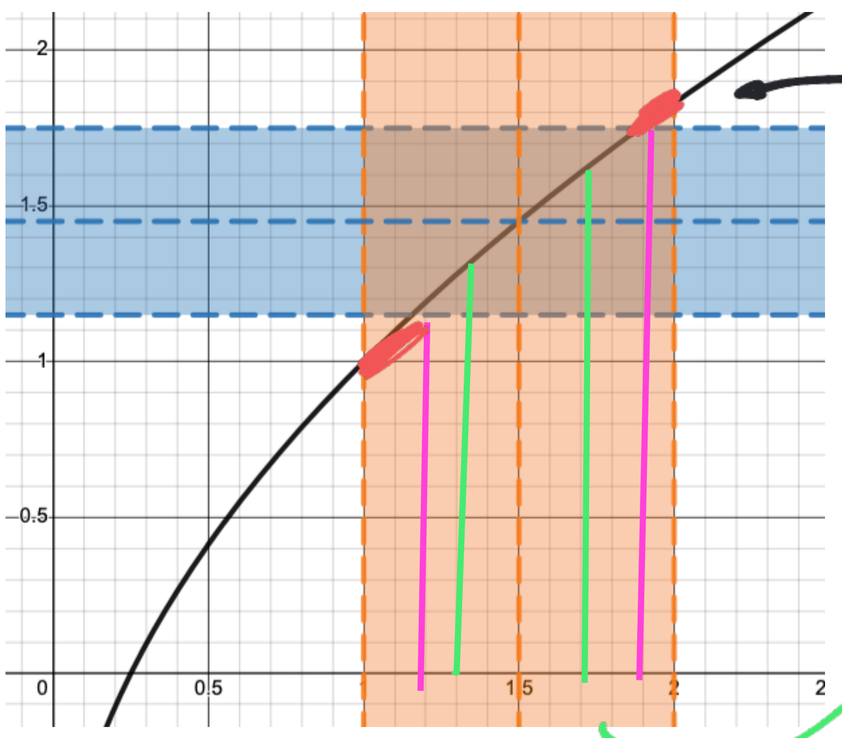
Given any $\varepsilon>0$ there exists $N>0$ such that if $x>N$ then $|f(x)-L|<\varepsilon$

Explain- A few words make it clear whether you understand

(3) (8 points)

(a) The graph below depicts $f(x) = 2\sqrt{x} - 1$ with $\varepsilon = 0.3$ and $\delta = 0.5$, $a = 1.5$, (as described in the definition of limit). Does this value of δ satisfy the definition for the given ε? No

(b) According to the graph, would $\delta = 0.2$ satisfy the defintion?(Explain) Yes

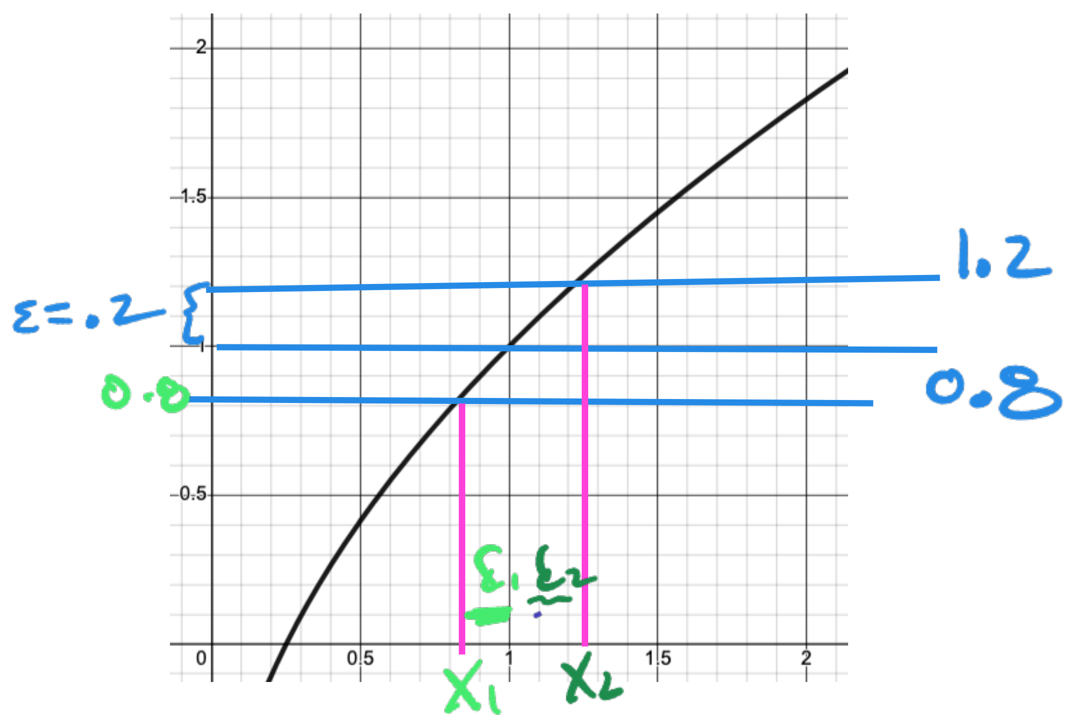


There are x values for which $|f(x)-L|$ is not $< \varepsilon$.

Just using graph....
green corresponds to $\delta = 2$

(c) Now suppose that for the same function but with $a = 1$, $\varepsilon = 0.2$. Find a value of δ that would satisfy the definition of limit. Show work.

(The graph is just shown for you to use if it helps)



Find x values where condition $|f(x)-L| < \varepsilon$ is met.

$\boxed{x_1}$

$f(x_1) = 0.8$

$2\sqrt{x_1} - 1 = 0.8$

$2\sqrt{x_1} = 1.8$

$\sqrt{x_1} = .9$

$x_1 = .9^2$

Distance δ_1 to $a = 1$

$\delta_1 = 1 - x_1$

$= 1 - .9^2 = .19$

$\boxed{x_2}$

$f(x_2) = 1.2$

$2\sqrt{x_2} - 1 = 1.2$

$2\sqrt{x_2} = 2.2$

$\sqrt{x_2} = 1.1$

$x_2 = 1.1^2$

$\delta_2 = x_2 - 1$

$= 1.1^2 - 1$

$= .21$

Answer: $\delta =$ .19

(smaller of δ_1, δ_2)

This is the largest value of δ that would work. Anything smaller works also.

(4) Evaluate and simplify the following limits if they exist (if the limit is ∞ or $-\infty$ say so.). No proof or detailed steps necessary, but do show work and use **proper notation.** Making a table of values or pluggin in one value is not an acceptable technique.(4 points each)

(a) $\displaystyle\lim_{x\to\frac{\pi}{3}} \frac{1}{\cos x - 4} = \underline{-2/7}$.

$$\frac{1}{\frac{1}{2}-4}$$

(b) $\displaystyle\lim_{x\to1^+} \frac{-2+3x}{x-1} = \underline{\infty}$

"$\frac{1}{0}$" $\Rightarrow$ vertical asymptote $\nearrow \infty$, $\searrow -\infty$

Positive or negative?

$\frac{+}{+} \to +\infty$

(c) $\displaystyle\lim_{x\to9} \frac{x-9}{\sqrt{x}-3} = \underline{6}$.

$$= \lim_{x\to9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$= \lim_{x\to9} \frac{(x-9)(\sqrt{x}+3)}{x-9}$$

$$= \lim_{x\to9} (\sqrt{x}+3) = 6$$

(d) $\displaystyle\lim_{x\to0^-}\left(\frac{1}{x}+\frac{1}{|x|}\right) = \underline{0}$

$x\to0^- \Rightarrow x<0 \Rightarrow |x| = -x$

$$\lim_{x\to0^-}\left(\frac{1}{x}+\frac{1}{|x|}\right) = \lim_{x\to0^-}\left(\frac{1}{x}+\frac{1}{-x}\right)$$

$$= \lim_{x\to0^-}(0) = 0$$

(e) $\displaystyle\lim_{x\to5} \frac{2x^2-9x-5}{x-5} = \underline{11}$.

$$= \lim_{x\to5} \frac{(x-5)(2x+1)}{x-5}$$

$$= \lim_{x\to5} (2x+1)$$

$$= 11$$

(f) $\displaystyle\lim_{x\to-\infty} \frac{3x}{\sqrt{x^2-5}} = \underline{-3}$

(detailed steps must be shown)

$$= \lim_{x\to-\infty} \frac{3x}{\sqrt{x^2(1-\frac{5}{x^2})}}$$

$$= \lim_{x\to-\infty} \frac{3x}{|x|(1-\frac{5}{x^2})}$$

since $x\to-\infty$, $x<0$, so $|x| = -x$

$$= \lim_{x\to-\infty} \frac{3x}{-x(1-\frac{5}{x^2})} = \lim_{x\to-\infty} \frac{-3}{1-5/x^2}$$

(5) Given If f(x)= $\begin{cases} \sqrt{11-x} & if\ x \le 2 \\ x^2 - 1 & if\ x > 2 \end{cases}$ find (8 points)

$\lim_{x \to 2^-} f(x) =$ 3 $\lim_{x \to 2^+} f(x) =$ 3

$\lim_{x \to 2} f(x) =$ 3 Is $f(x)$ continuous at x=2? yes

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(6) For what values of x are the following functions continuous? Show work. (4 points each)

a) $f(x) = \dfrac{4x}{\tan x + 1}$

Conts. for all real values except:

denom $\neq 0 \Rightarrow \tan x + 1 \neq 0$

$\tan x \neq -1$

$\boxed{x \neq \frac{3\pi}{4} + \pi k}$

AND

since tanx domain

$\boxed{x \neq \frac{\pi}{2} + \pi k}$

b) $f(x) = \sqrt{x^2 - x - 12}$

$= \sqrt{(x-4)(x-3)}$

Conts on domain

radicand ≥ 0

$(x-4)(x+3) \geq 0$

+ − +

−3 4

Conts for

$\boxed{(-\infty, -3] \cup [4, \infty)}$

(7) The table shows the position of a cyclist. (8 points)

t (seconds)	0	1	3	4	5	7
s (feet)	0	1.5	6.5	13.5	24.0	37.5

(Express answers using appropriate units.)

a) Find the average velocity over the time period [1,4].

$$\underset{[1,4]}{\text{Ave Velocity}} = \frac{s(4)-s(1)}{4-1} = \frac{13.5-1.5}{3} = \boxed{4 \text{ ft/sec}}$$

b) Estimate the instantaneous velocity of the cyclist at t=1 as accurately as possible.

$$\underset{[0,1]}{\text{Ave. Velocity}} = \frac{s(1)-s(0)}{1-0} = \frac{1.5}{1} = 1.5 \text{ ft/sec}$$

$$\underset{[1,3]}{\text{Ave Velocity}} \quad \frac{s(3)-s(1)}{3-1} = \frac{6.5-1.5}{2} = 2.5 \text{ ft/sec}$$

For better estimate, average those two

$$\text{Instantaneous velocity at } t=1 \approx \frac{1.5+2.5}{2} = \boxed{2 \text{ ft/sec}}$$

(8) Given $f(x)=x^2-2$	(20 points)

a) Use an appropriate form of the definition (methods we have used in this class, no short cuts) of the derivative to compute $f'(a)$.

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{x^2-2-(a^2-2)}{x-a}=\lim_{x\to a}\frac{x^2-a^2}{x-a}$$

$$=\lim_{x\to a}\frac{(x-a)(x+a)}{x-a}=\lim_{x\to a}(x+a)=2a$$

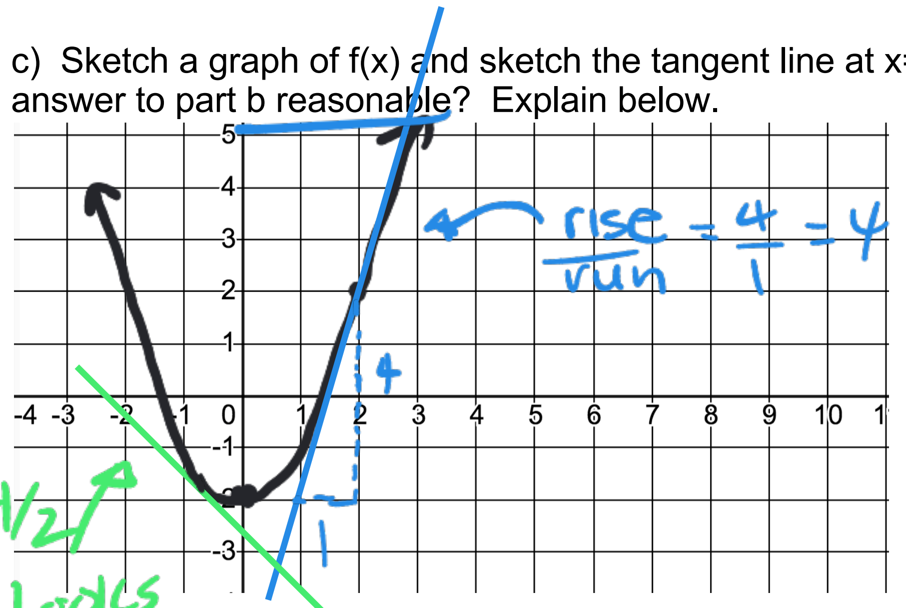
ANS

$f'(a)=$ 2a

b) Use the results of part (a) to find the slope of the tangent line at x = 0, 2, and 3

slope at x= 0 0 slope at x=2 4 6

c) Sketch a graph of f(x) and sketch the tangent line at x= 2. Based on your graph, Is your answer to part b reasonable? Explain below.



at x=-1/2
slope looks like -1, reasonable

d) Find the equation of the tangent line at x= 2.

point $(2, f(2))=(2,2)$
$m=f'(2)=4$

Tangent line at x= 2

$y-2=4(x-2)$ or $y=4x-6$

e) Using part (a), find a value for a such that $f'(a)=-1$. Is this answer reasonable according to your graph?

$a=$ -1/2

$f'(a)=-1$
$2a=-1$
$a=-1/2$